Bulg. J. Phys. **30** (2015) 1–14

Metric-Independent Volume-Forms in Gravity and Cosmology *

E. Guendelman[1], **E. Nissimov**[2], **S. Pacheva**[2]

[1]Department of Physics, Ben-Gurion Univ. of the Negev, Beer-Sheva 84105, Israel

[2]Institute of Nuclear Research and Nuclear Energy, Bulg. Acad. Sci., Sofia 1784, Bulgaria

Received

Abstract. Employing alternative spacetime volume-forms (generally-covariant integration measure densities) independent of the pertinent Riemannian spacetime metric have profound impact in general relativity. Although formally appearing as "pure-gauge" dynamical degrees of freedom they trigger a number of remarkable physically important phenomena such as: (i) new mechanism of dynamical generation of cosmological constant; (ii) new type of "quintessential inflation" scenario in cosmology; (iii) non-singular initial "emergent universe" phase of cosmological evolution preceding the inflationary phase; (iv) new mechanism of dynamical spontaneous breakdown of supersymmetry in supergravity; (v) gravitational electrovacuum "bags". We study in some detail the properties, together with their canonical Hamiltonian formulation, of a class of generalized gravity-matter models built with two independent non-Riemannian volume-forms and discuss their implications in cosmology.

PACS number: 04.50.Kd,11.30.Qc,98.80.Bp

1 Introduction

Alternative spacetime volume-forms (generally-covariant integration measure densities) independent on the Riemannian metric on the pertinent spacetime manifold have profound impact in field theory models with general coordinate reparametrization invariance – general relativity and its extensions, strings and (higher-dimensional) membranes.

Although formally appearing as "pure-gauge" dynamical degrees of freedom the non-Riemannian volume-form fields trigger a number of remarkable physically important phenomena. Among the principal new phenomena are:

*Invited talk at the Memorial *"Matey Mateev Symposium"*, April 2015, *https://indico.cern.ch/event/359644/*. E.N. and S.P. express their sincere gratitude and appreciation to the late Acad. Prof. M. Mateev – one of their first university teachers.

- (i) new mechanism of dynamical generation of cosmological constant;
- (ii) new type of ”quintessential inflation” scenario in cosmology describing both the “early” and “late” universe in terms of a single scalar field;
- (iii) non-singular initial phase of cosmological evolution – “no Big-Bang” “emergent universe” – preceding the inflationary phase;
- (iv) new mechanism of dynamical spontaneous breakdown of supersymmetry in supergravity;
- (v) Coupling of non-Riemannian volume-form gravity-matter theories to a special non-standard kind of nonlinear gauge system containing the square-root of standard Maxwell/Yang-Mills Lagrangian yields charge confinement/deconfinement phases associated with gravitational electrovacuum “bags”.

Properties (i)-(iii) are discussed in more details in what follows.

The ideas of the present formalism rely substantially on a series of previous papers [1] (for recent developments, see Refs. [2]), where a new class of generally-covariant (non-supersymmetric) field theory models including gravity – called “two-measure theories” (TMT) was proposed based on the principal proposal to employ an alternative volume form (volume element or generally-covariant integration measure) on the spacetime manifold in the pertinent Lagrangian action. TMT appear to be promising candidates for resolution of various problems in modern cosmology: the *dark energy* and *dark matter* problems, the fifth force problem, etc.

In standard generally-covariant theories (with action $S = \int d^D x \sqrt{-g}\mathcal{L}$) the Riemannian spacetime volume-form, *i.e.*, the integration measure density is given by $\sqrt{-g}$, where $g \equiv \det \|g_{\mu\nu}\|$ is the determinant of the corresponding Riemannian metric $g_{\mu\nu}$. $\sqrt{-g}$ transforms as scalar density under general coordinate reparametrizations.

There is *no a priori* any obstacle to employ instead of $\sqrt{-g}$ another alternative non-Riemannian volume element given by the following *non-Riemannian* integration measure density:

$$\Phi(B) \equiv \frac{1}{(D-1)!}\varepsilon^{\mu_1 \ldots \mu_D}\, \partial_{\mu_1} B_{\mu_2 \ldots \mu_D} \; . \tag{1}$$

Here $B_{\mu_1 \ldots \mu_{D-1}}$ is an auxiliary rank $(D-1)$ antisymmetric tensor gauge field, which will turn out to be pure-gauge degree of freedom. $\Phi(B)$ – the dual field-strength of $B_{\mu_1 \ldots \mu_{D-1}}$ – similarly transforms as scalar density under general coordinate reparametrizations like $\sqrt{-g}$.

An important property of the present formalism is that the non-Riemannian measure density $\Phi(B)$ becomes *on-shell* proportional to the standard Riemannian

one $\sqrt{-g}$ (see Eq.(9) below), *i.e.*, the physical meaning of $\Phi(B)$ as a measure is preserved.

In the next Section 2 we describe in some detail the construction within the Lagrangian formalism of a new class of generalized gravity-matter theories built in terms of two different non-Riemannian volume-forms and derive the effective Lagrangian in the physical "Einstein frame". In Section 3 we provide a general canonical Hamiltonian treatment of gravity-matter theories with non-Riemannian volume-forms and elucidate the physical meaning of the auxiliary volume-form fields. Section 4 is devoted to discussion of the cosmological implications of the above class of generalized gravity-matter theories with two non-Riemannian volume-forms. The central result here is the derivation of an effective scalar ("inflaton") potential with *two infinitely large flat regions* with vastly different energy scales. In Section 5 we briefly describe the construction of the "emergent universe" solution as a non-singular ("no Big-Bang") initial phase of cosmological evolution preceding the inflationary phase.

2 Gravity-Matter Theories with Two Non-Riemannian Volume-Forms

Let us now consider modified-measure gravity-matter theories constructed in terms of two different non-Riemannian volume-forms (employing first-order Palatini formalism, and using units where $G_{\rm Newton} = 1/16\pi$) [3]:

$$S = \int d^4 x\, \Phi_1(A)\Big[R + L^{(1)}\Big] + \int d^4 x\, \Phi_2(B)\Big[L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}}\Big] \; . \quad (2)$$

Here and below the following notations are used:

- $\Phi_1(A)$ and $\Phi_2(B)$ are two independent non-Riemannian volume-forms:

$$\Phi_1(A) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu A_{\nu\kappa\lambda} \quad , \quad \Phi_2(B) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu B_{\nu\kappa\lambda} \; . \quad (3)$$

- $\Phi(H)$ is the dual field-strength of a third auxiliary gauge field $H_{\mu\nu\lambda}$:

$$\Phi(H) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu H_{\nu\kappa\lambda} \; , \quad (4)$$

  whose presence is essential for the consistency of (2).

- $R = g^{\mu\nu} R_{\mu\nu}(\Gamma)$ and $R_{\mu\nu}(\Gamma)$ are the scalar curvature and the Ricci tensor in the first-order (Palatini) formalism, where the affine connection $\Gamma^\mu_{\nu\lambda}$ is *a priori* independent of the metric $g_{\mu\nu}$. In the second action term in (2) we have added a R^2 gravity term (again in the Palatini form). The gravity model $R+R^2$ within the second order formalism was the first inflationary model originally proposed in Ref. [4].

- $L^{(1,2)}$ denote two different Lagrangians of a single scalar matter field ("dilaton" or "inflaton") of the form:

$$L^{(1)} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \quad , \quad V(\varphi) = f_1 \exp\{-\alpha\varphi\} \; , \quad (5)$$

$$L^{(2)} = -\frac{b}{2} e^{-\alpha\varphi} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + U(\varphi) \quad , \quad U(\varphi) = f_2 \exp\{-2\alpha\varphi\} \; , \quad (6)$$

  where α, f_1, f_2 are dimensionful positive parameters, whereas b is a dimensionless one.

The action (2) possesses a *global Weyl-scale invariance*:

$$g_{\mu\nu} \to \lambda g_{\mu\nu} \; , \; \Gamma^\mu_{\nu\lambda} \to \Gamma^\mu_{\nu\lambda} \; , \; \varphi \to \varphi + \frac{1}{\alpha} \ln \lambda \; , \quad (7)$$
$$A_{\mu\nu\kappa} \to \lambda A_{\mu\nu\kappa} \; , \; B_{\mu\nu\kappa} \to \lambda^2 B_{\mu\nu\kappa} \; , \; H_{\mu\nu\kappa} \to H_{\mu\nu\kappa} \; .$$

The equations of motion w.r.t. affine connection $\Gamma^\mu_{\nu\lambda}$ yield the following solution for the latter as a Levi-Civita connection:

$$\Gamma^\mu_{\nu\lambda} = \Gamma^\mu_{\nu\lambda}(\bar{g}) = \frac{1}{2} \bar{g}^{\mu\kappa} \left(\partial_\nu \bar{g}_{\lambda\kappa} + \partial_\lambda \bar{g}_{\nu\kappa} - \partial_\kappa \bar{g}_{\nu\lambda}\right) \; , \quad (8)$$

corresponding to the Weyl-rescaled metric $\bar{g}_{\mu\nu}$:

$$\bar{g}_{\mu\nu} = (\chi_1 + 2\epsilon\chi_2 R) g_{\mu\nu} \quad , \quad \chi_1 \equiv \frac{\Phi_1(A)}{\sqrt{-g}} \quad , \quad \chi_2 \equiv \frac{\Phi_2(B)}{\sqrt{-g}} \; . \quad (9)$$

Transition from the original metric $g_{\mu\nu}$ to $\bar{g}_{\mu\nu}$ realizes the passage to the physical *"Einstein-frame"*, where the gravity equations of motion acquire the standard form of Einstein's equations:

$$R_{\mu\nu}(\bar{g}) - \frac{1}{2} \bar{g}_{\mu\nu} R(\bar{g}) = \frac{1}{2} T^{\rm eff}_{\mu\nu} \quad (10)$$

with an appropriate *effective matter energy-momentum tensor* given in terms of an *effective Einstein-frame matter Lagrangian* $L_{\rm eff}$ (see (21) below).

Variation of the action (2) w.r.t. auxiliary tensor gauge fields $A_{\mu\nu\lambda}$, $B_{\mu\nu\lambda}$ and $H_{\mu\nu\lambda}$ yields the equations:

$$\partial_\mu \Bigl[R + L^{(1)}\Bigr] = 0 \; , \; \partial_\mu \Bigl[L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}}\Bigr] = 0 \; , \; \partial_\mu \Bigl(\frac{\Phi_2(B)}{\sqrt{-g}}\Bigr) = 0 \; , \quad (11)$$

whose solutions read:

$$\frac{\Phi_2(B)}{\sqrt{-g}} \equiv \chi_2 = {\rm const} \quad , \quad R + L^{(1)} = -M_1 = {\rm const} \; ,$$
$$L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}} = -M_2 = {\rm const} \; . \quad (12)$$

Here M_1 and M_2 are arbitrary dimensionful and χ_2 arbitrary dimensionless integration constants.

The first integration constant χ_2 in (12) preserves global Weyl-scale invariance (7) whereas the appearance of the second and third integration constants $M_1,\, M_2$ signifies *dynamical spontaneous breakdown* of global Weyl-scale invariance under (7) due to the scale non-invariant solutions (second and third ones) in (12).

It is very instructive to elucidate the physical meaning of the three arbitrary integration constants $M_1,\, M_2,\, \chi_2$ from the point of view of the canonical Hamiltonian formalism. Namely, $M_1,\, M_2,\, \chi_2$ are identified as conserved Dirac-constrained canonical momenta conjugated to (certain components of) the auxiliary maximal rank antisymmetric tensor gauge fields $A_{\mu\nu\lambda}, B_{\mu\nu\lambda}, H_{\mu\nu\lambda}$ entering the original non-Riemannian volume-form action (2) (for details, see next Section 3 below).

Varying (2) w.r.t. $g_{\mu\nu}$ and using relations (12) we have:

$$\chi_1 \Big[R_{\mu\nu} + \frac{1}{2}\Big(g_{\mu\nu}L^{(1)} - T^{(1)}_{\mu\nu}\Big)\Big] - \frac{1}{2}\chi_2 \Big[T^{(2)}_{\mu\nu} + g_{\mu\nu}\left(\epsilon R^2 + M_2\right) - 2R\, R_{\mu\nu}\Big] = 0 \; , \tag{13}$$

where χ_1 and χ_2 are defined in (9), and $T^{(1,2)}_{\mu\nu}$ are the energy-momentum tensors of the scalar field Lagrangians with the standard definitions:

$$T^{(1,2)}_{\mu\nu} = g_{\mu\nu} L^{(1,2)} - 2\frac{\partial}{\partial g^{\mu\nu}} L^{(1,2)} \; . \tag{14}$$

Taking the trace of Eqs.(13) and using again second relation (12) we solve for the scale factor χ_1:

$$\chi_1 = 2\chi_2 \frac{T^{(2)}/4 + M_2}{L^{(1)} - T^{(1)}/2 - M_1} \; , \tag{15}$$

where $T^{(1,2)} = g^{\mu\nu} T^{(1,2)}_{\mu\nu}$.

Using second relation (12) Eqs.(13) can be put in the Einstein-like form:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{2} g_{\mu\nu}\Big(L^{(1)} + M_1\Big) + \frac{1}{2\Omega}\Big(T^{(1)}_{\mu\nu} - g_{\mu\nu}L^{(1)}\Big) + \frac{\chi_2}{2\chi_1\Omega}\Big[T^{(2)}_{\mu\nu} + g_{\mu\nu}\Big(M_2 + \epsilon(L^{(1)} + M_1)^2\Big)\Big] \; , \tag{16}$$

where:

$$\Omega = 1 - \frac{\chi_2}{\chi_1} 2\epsilon \Big(L^{(1)} + M_1\Big) \; . \tag{17}$$

Let us note that (9), upon taking into account second relation (12) and (17), can be written as:

$$\bar{g}_{\mu\nu} = \chi_1 \Omega\, g_{\mu\nu} \; . \tag{18}$$

Now, we can bring Eqs.(16) into the standard form of Einstein equations for the rescaled metric $\bar{g}_{\mu\nu}$ (18), *i.e.*, the Einstein-frame equations:

$$R_{\mu\nu}(\bar{g}) - \frac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) = \frac{1}{2}T^{\rm eff}_{\mu\nu} \tag{19}$$

with energy-momentum tensor corresponding according to the definition (14):

$$T^{\rm eff}_{\mu\nu} = g_{\mu\nu}L_{\rm eff} - 2\frac{\partial}{\partial g^{\mu\nu}}L_{\rm eff} \tag{20}$$

to the following effective (Einstein-frame) scalar field Lagrangian of non-canonical "k-essence" (kinetic quintessence) type [5] ($X \equiv -\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$ denotes the scalar kinetic term):

$$L_{\rm eff} = A(\varphi)X + B(\varphi)X^2 - U_{\rm eff}(\varphi) \; , \tag{21}$$

where (recall $V = f_1 e^{-\alpha\varphi}$ and $U = f_2 e^{-2\alpha\varphi}$):

$$A(\varphi) \equiv 1 + \Big[\frac{1}{2}be^{-\alpha\varphi} - \epsilon(V - M_1)\Big]\frac{V - M_1}{U + M_2 + \epsilon(V - M_1)^2} \; , \tag{22}$$

$$B(\varphi) \equiv \chi_2\frac{\epsilon\Big[U + M_2 + (V - M_1)be^{-\alpha\varphi}\Big] - \frac{1}{4}b^2e^{-2\alpha\varphi}}{U + M_2 + \epsilon(V - M_1)^2} \; , \tag{23}$$

$$U_{\rm eff}(\varphi) \equiv \frac{(V - M_1)^2}{4\chi_2\Big[U + M_2 + \epsilon(V - M_1)^2\Big]} \; . \tag{24}$$

3 Canonical Hamiltonian Treatment of Gravity-Matter Theories with Non-Riemannian Volume-Forms

Here we will briefly discuss the application of the canonical Hamiltonian formalism to the new gravity-matter model based on two non-Riemannian spacetime volume-forms (2). In order to elucidate the proper physical meaning of the arbitrary integration constants $\chi_2,\, M_1,\, M_2$ (12) encountered within the Lagrangian formalism's treatment of (2) it is sufficient to concentrate only on the canonical Hamiltonian structure related to the auxiliary maximal rank antisymmetric tensor gauge fields $A_{\mu\nu\lambda}, B_{\mu\nu\lambda}, H_{\mu\nu\lambda}$ and their respective conjugate momenta.

For convenience let us introduce the following short-hand notations for the field-strengths (3), (4) of the auxiliary 3-index antisymmetric gauge fields (the dot indicating time-derivative):

$$\Phi_1(A) = \dot{A} + \partial_i A^i \quad , \quad A = \frac{1}{3!}\varepsilon^{ijk}A_{ijk} \;\; , \;\; A^i = -\frac{1}{2}\varepsilon^{ijk}A_{0jk} \; , \tag{25}$$

$$\Phi_2(B) = \dot{B} + \partial_i B^i \quad , \quad B = \frac{1}{3!}\varepsilon^{ijk}B_{ijk} \;\; , \;\; B^i = -\frac{1}{2}\varepsilon^{ijk}B_{0jk} \; , \tag{26}$$

$$\Phi(H) = \dot{H} + \partial_i H^i \quad , \quad H = \frac{1}{3!}\varepsilon^{ijk}H_{ijk} \;\; , \;\; H^i = -\frac{1}{2}\varepsilon^{ijk}H_{0jk} \; , \tag{27}$$

Also we will use the short-hand notation:

$$\widetilde{L}^{(1)}(u,\dot{u}) \equiv R + L^{(1)} \quad , \quad \widetilde{L}^{(2)}(u,\dot{u}) \equiv L^{(2)} + \epsilon R^2 \; , \tag{28}$$

where $L^{(1,2)}$ are as in (5)-(6) and where $(u,\dot{u})$ collectively denote the set of the basic gravity-matter canonical variables $(u) = \big(g_{\mu\nu}, \varphi, A_\mu\big)$ and their respective velocities.

For the pertinent canonical momenta conjugated to (25)-(27) we have:

$$\pi_A = \widetilde{L}_1(u,\dot{u}) \;\; , \;\; \pi_B = \widetilde{L}^{(2)}(u,\dot{u}) + \frac{1}{\sqrt{-g}}(\dot{H} + \partial_i H^i) \; , \\ \pi_H = \frac{1}{\sqrt{-g}}(\dot{B} + \partial_i B^i) \; , \tag{29}$$

and:

$$\pi_{A^i} = 0 \quad , \quad \pi_{B^i} = 0 \quad , \quad \pi_{H^i} = 0 \; . \tag{30}$$

The latter imply that A^i, B^i, H^i will in fact appear as Lagrange multipliers for certain first-class Hamiltonian constraints (see Eqs.(34)-(35) below). For the canonical momenta conjugated to the basic gravity-matter canonical variables we have (using last relation (29)):

$$p_u = (\dot{A} + \partial_i A^i)\frac{\partial}{\partial\, \dot{u}}\widetilde{L}_1(u,\dot{u}) + \pi_H \sqrt{-g}\frac{\partial}{\partial\, \dot{u}} L^{(2)}(u,\dot{u}) \; . \tag{31}$$

Now, relations (29) and (31) allow us to obtain the velocities $\dot{u}$, $\dot{A}$, $\dot{B}$, $\dot{H}$ as functions of the canonically conjugate momenta $\dot{u}{=}\dot{u}\,(u, p_u, \pi_A, \pi_B, \pi_H)$ *etc.* (modulo some Dirac constraints among the basic gravity-matter variables due to general coordinate and gauge invariances). Taking into account (29)-(30) (and the short-hand notations (25)-(28)) the canonical Hamiltonian corresponding to (2):

$$\mathcal{H} = p_u\, \dot{u} + \pi_A\, \dot{A} + \pi_B\, \dot{B} + \pi_H\, \dot{H} - (\dot{A} + \partial_i A^i)\widetilde{L}_1(u,\dot{u}) \\ - \pi_H \sqrt{-g}\Big[\widetilde{L}^{(2)}(u,\dot{u}) + \frac{1}{\sqrt{-g}}(\dot{H} + \partial_i H^i)\Big] \tag{32}$$

acquires the following form as function of the canonically conjugated variables (here $\dot{u}{=}\dot{u}\,(u, p_u, \pi_A, \pi_B, \pi_H)$):

$$\mathcal{H} = p_u\, \dot{u} - \pi_H \sqrt{-g}\widetilde{L}^{(2)}(u,\dot{u}) \\ + \sqrt{-g}\pi_H \pi_B - \partial_i A^i \pi_A - \partial_i B^i \pi_B - \partial_i H^i \pi_H \; . \tag{33}$$

From (33) we deduce that indeed A^i, B^i, H^i are Lagrange multipliers for the first-class Hamiltonian constraints:

$$\partial_i \pi_A = 0 \;\; \to \;\; \pi_A = -M_1 = \mathrm{const} \; , \tag{34}$$

and similarly:

$$\pi_B = -M_2 = \mathrm{const} \quad , \quad \pi_H = \chi_2 = \mathrm{const} \; , \tag{35}$$

which are the canonical Hamiltonian counterparts of Lagrangian constraint equations of motion (12).

Thus, the canonical Hamiltonian treatment of (2) reveals the meaning of the auxiliary 3-index antisymmetric tensor gauge fields $A_{\mu\nu\lambda}, \, B_{\mu\nu\lambda}, \, H_{\mu\nu\lambda}$ – building blocks of the non-Riemannian spacetime volume-form formulation of the modified gravity-matter model (2). Namely, the canonical momenta $\pi_A, \, \pi_B, \, \pi_H$ conjugated to the "magnetic" parts A, B, H (25)-(27) of the auxiliary 3-index antisymmetric tensor gauge fields are constrained through Dirac first-class constraints (34)-(35) to be constants identified with the arbitrary integration constants $\chi_2, \, M_1, \, M_2$ (12) arising within the Lagrangian formulation of the model. The canonical momenta $\pi^i_A, \, \pi^i_B, \, \pi^i_H$ conjugated to the "electric" parts A^i, B^i, H^i (25)-(27) of the auxiliary 3-index antisymmetric tensor gauge field are vanishing (30) which makes the latter canonical Lagrange multipliers for the above Dirac first-class constraints.

4 Implications for Cosmology

The most remarkable feature of the effective scalar potential $U_{\rm eff}(\varphi)$ (24) is the existence of the following two *infinitely large flat regions* as function of φ:

- *(-) flat region* – for large negative values of φ:

$$U_{\rm eff}(\varphi) \simeq U_{(-)} \equiv \frac{f_1^2/f_2}{4\chi_2(1+\epsilon f_1^2/f_2)} \; , \tag{36}$$

- (+) flat region – for large positive values of φ:

$$U_{\rm eff}(\varphi) \simeq U_{(+)} \equiv \frac{M_1^2/M_2}{4\chi_2(1+\epsilon M_1^2/M_2)} \; , \tag{37}$$

The qualitative shape of $U_{\rm eff}(\varphi)$ (24) is depicted on Figs.1 and 2.

From the expression for $U_{\rm eff}(\varphi)$ (24) and the Figures 1 and 2 we deduce that we have an *explicit realization of quintessential inflation scenario* [6]: continuously connecting an inflationary phase of the universe's evolution corresponding to the (most of the) $(-)$-flat region to a slowly accelerating "present-day" universe corresponding to the $(+)$-flat region through the evolution of a single scalar field.

The flat regions (36) and (37) correspond to the evolution of *early* and the *late* universe, respectively, provided we choose the ratio of the coupling constants

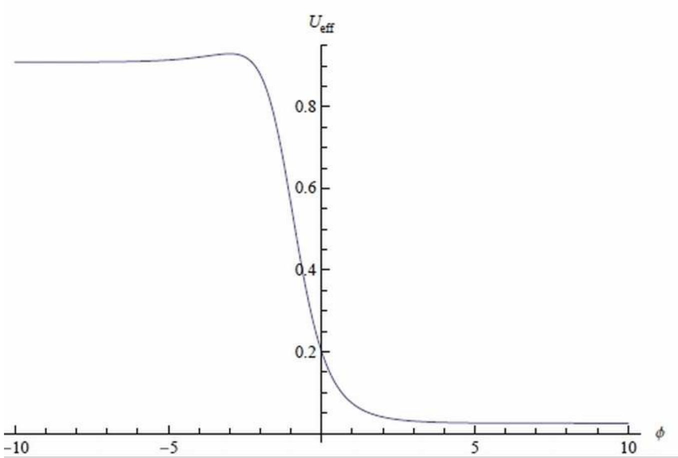

Figure 1. Qualitative shape of the effective scalar potential $U_{\rm eff}$ (24) as function of φ for $M_1 < 0$.

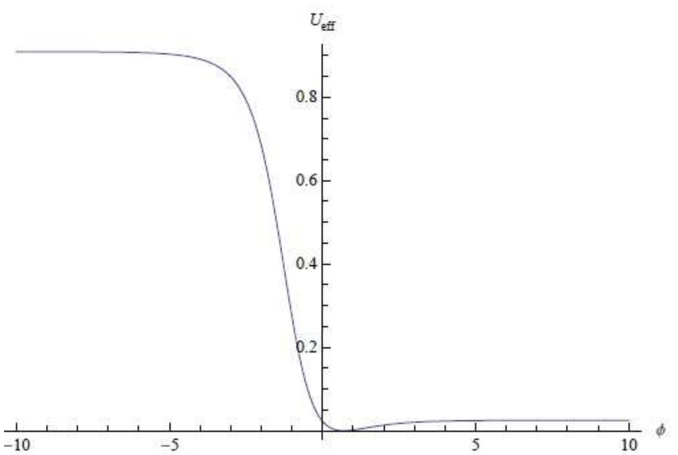

Figure 2. Qualitative shape of the effective scalar potential $U_{\rm eff}$ (24) as function of φ for $M_1 > 0$.

in the original scalar potentials versus the ratio of the scale-symmetry breaking integration constants to obey the following strong inequality:

$$\frac{f_1^2/f_2}{1+\epsilon f_1^2/f_2} \gg \frac{M_1^2/M_2}{1+\epsilon M_1^2/M_2}\,, \tag{38}$$

which makes the *vacuum energy density of the early universe* $U_{(-)}$ (36) much bigger than that of the late universe $U_{(+)}$ (37)).

The inequality (38) is equivalent to the requirements:

$$\frac{f_1^2}{f_2} \gg \frac{M_1^2}{M_2} \quad , \quad |\epsilon| \frac{M_1^2}{M_2} \ll 1 \; . \tag{39}$$

If we choose the scales $|M_1| \sim M_{EW}^4$ and $M_2 \sim M_{Pl}^4$ [7], where M_{EW}, M_{Pl} are the electroweak and Planck scales, respectively, we are then naturally led to a very small vacuum energy density:

$$U_{(+)} \sim M_{EW}^8 / M_{Pl}^4 \sim 10^{-120} M_{Pl}^4 \; , \tag{40}$$

which is the right order of magnitude for the present epoch's vacuum energy density.

On the other hand, if we take the order of magnitude of the coupling constants in the effective potential $f_1 \sim f_2 \sim (10^{-2} M_{Pl})^4$, then the order of magnitude of the vacuum energy density of the early universe becomes:

$$U_{(-)} \sim f_1^2 / f_2 \sim 10^{-8} M_{Pl}^4 \; , \tag{41}$$

which conforms to the Planck Collaboration data [8] implying the energy scale of inflation to be of order $10^{-2} M_{Pl}$.

5 "Emergent universe"

Within the present gravity-matter theory with two non-Riemannian spacetime volume-forms we find explicit cosmological solution of the Einstein-frame system with effective scalar field Lagrangian (21)-(24) describing an epoch of a *non-singular creation of the universe* – "emergent universe" [9], preceding the inflationary phase.

The starting point are the Friedman equations [10]:

$$\frac{\ddot{a}}{a} = -\frac{1}{12}(\rho + 3p) \quad , \quad H^2 + \frac{K}{a^2} = \frac{1}{6}\rho \quad , \quad H \equiv \frac{\dot{a}}{a} \; , \tag{42}$$

describing the universe' evolution. Here:

$$\rho = \frac{1}{2} A(\varphi)\, \dot{\varphi}^2 + \frac{3}{4} B(\varphi)\, \dot{\varphi}^4 + U_{\rm eff}(\varphi) \; , \tag{43}$$

$$p = \frac{1}{2} A(\varphi)\, \dot{\varphi}^2 + \frac{1}{4} B(\varphi)\, \dot{\varphi}^4 - U_{\rm eff}(\varphi) \tag{44}$$

are the energy density and pressure of the scalar field $\varphi = \varphi(t)$, H is the Hubble parameter and K denotes the Gaussian curvature of the spacial section in the Friedman-Lemaitre-Robertson-Walker metric [10]:

$$ds^2 = -dt^2 + a^2(t) \Big[\frac{dr^2}{1 - K r^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \Big] \; . \tag{45}$$

"Emergent universe" is defined as a solution of the Friedman Eqs.(42) subject to the condition on the Hubble parameter H:

$$H = 0 \quad \to \quad a(t) = a_0 = \mathrm{const}\,, \quad \rho + 3p = 0 \quad , \quad \frac{K}{a_0^2} = \frac{1}{6}\rho \; (= \mathrm{const}) \; , \tag{46}$$

with ρ and p as in (43)-(44). Here $K = 1$ ("Einstein universe").

The "emergent universe" condition (46) implies that the φ-velocity $\dot{\varphi} \equiv \dot{\varphi}_0$ is time-independent and satisfies the bi-quadratic algebraic equation:

$$\frac{3}{2} B_{(-)} \, \dot{\varphi}_0^4 + 2 A_{(-)} \, \dot{\varphi}_0^2 - 2 U_{(-)} = 0 \; , \tag{47}$$

where $A_{(-)}, \, B_{(-)}, \, U_{(-)}$ are the limiting values on the $(-)$ flat region of $A(\varphi), \; B(\varphi), \; U_{\mathrm{eff}}(\varphi)$ (22)-(24).

The solution of Eq.(47) reads:

$$\dot{\varphi}_0^2 = - \frac{2}{3 B_{(-)}} \Bigl[A_{(-)} \mp \sqrt{A_{(-)}^2 + 3 B_{(-)} U_{(-)}} \Bigr] \; . \tag{48}$$

and, thus, the "emergent universe" is characterized with *finite initial* Friedman factor and density:

$$a_0^2 = \frac{6K}{\rho_0} \quad , \quad \rho_0 = \frac{1}{2} A_{(-)} \, \dot{\varphi}_0^2 + \frac{3}{4} B_{(-)} \, \dot{\varphi}_0^4 + U_{(-)} \; , \tag{49}$$

with $\dot{\varphi}_0^2$ as in (48).

Analysis of stability of the "emergent universe" solution (49) yields a harmonic oscillator type equation for the perturbation of the Friedman factor δa:

$$\delta \, \ddot{a} + \omega^2 \delta a = 0 \quad , \quad \omega^2 \equiv \frac{2}{3} \rho_0 \, \frac{\sqrt{A_{(-)}^2 + 3 B_{(-)} U_{(-)}}}{A_{(-)} - 2 \sqrt{A_{(-)}^2 + 3 B_{(-)} U_{(-)}}} \; . \tag{50}$$

Thus stability condition $\omega^2 > 0$ leads to the following constraint on the coupling parameters:

$$\max\Bigl\{ -2 \, , \, -8 \bigl(1 + 3\epsilon f_1^2/f_2\bigr) \Bigl[1 - \sqrt{1 - \frac{1}{4\bigl(1 + 3\epsilon f_1^2/f_2\bigr)}} \Bigr] \Bigr\} < b \frac{f_1}{f_2} < -1 \; . \tag{51}$$

Since the ratio $\frac{f_1^2}{f_2}$ proportional to the height of the $(-)$ flat region of the effective scalar potential, *i.e.*, the vacuum energy density in the early universe, must be large (cf. (38)), we find that the lower end of the interval in (51) is very close to the upper end, *i.e.*, $b \frac{f_1}{f_2} \simeq -1$.

From Eqs.(48)-(49) we obtain an inequality satisfied by the initial energy density ρ_0 in the emergent universe:

$$U_{(-)} < \rho_0 < 2U_{(-)} \; , \tag{52}$$

which together with the estimate of the order of magnitude for $U_{(-)}$ (41) implies order of magnitude for the initial Friedman factor:

$$a_0^2 \sim 10^{-8} K M_{Pl}^{-2} \tag{53}$$

(recall K is the Gaussian curvature of the spacial section).

6 Conclusions

- Non-Riemannian volume-form formalism in gravity/matter theories (*i.e.*, employing alternative non-Riemannian reparametrization covariant integration measure densities on the spacetime manifold) naturally generates a *dynamical cosmological constant* as an arbitrary dimensionful integration constant.
- Employing two different non-Riemannian volume-forms leads to the construction of a new class of gravity-matter models, which produce an effective scalar potential with *two infinitely large flat regions*. This allows for a unified description of both early universe inflation as well as of present dark energy epoch.
- A remarkable feature is the existence of a stable initial phase of *non-singular* universe creation preceding the inflationary phase – "emergent universe" without "Big-Bang".

Further very interesting features of gravity-matter theories built with non-Riemannian spacetime volume-forms include:

- Within non-Riemannian-modified-measure minimal $N = 1$ supergravity the dynamically generated cosmological constant triggers spontaneous supersymmetry breaking and mass generation for the gravitino (supersymmetric Brout-Englert-Higgs effect) [11]. Applying the same non-Riemannian volume-form formalism to anti-de Sitter supergravity allows to produce simultaneously a very large physical gravitino mass and a very small *positive* observable cosmological constant [11] in accordance with modern cosmological scenarios for slowly expanding universe of the present epoch [12].
- Adding interaction with a special nonlinear ("square-root" Maxwell) gauge field (known to describe charge confinement in flat spacetime) produces various phases with different strength of confinement and/or with

deconfinement, as well as gravitational electrovacuum "bags" partially mimicking the properties of *MIT bags* and solitonic constituent quark models (for details, see [13]).

Acknowledgments

We gratefully acknowledge support of our collaboration through the academic exchange agreement between the Ben-Gurion University and the Bulgarian Academy of Sciences. E.N. and S.P. are supported by Bulgarian NSF Grant DFNI T02/6. S.P. and E.N. have received partial support from COST actions MP-1210 and MP-1405, respectively.

References

[1] E.I. Guendelman (1999) *Mod. Phys. Lett.* **A14** 1043-1052 (*arXiv:gr-qc/9901017*); E. Guendelman, A. Kaganovich (1999) *Phys. Rev.* **D60** 065004 (*arXiv:gr-qc/9905029*); E. Guendelman, O. Katz (2003) *Class. Quant. Grav.* **20** 1715-1728 (*arXiv:gr-qc/0211095*).

[2] E.I. Guendelman, P. Labrana (2013) *Int. J. Mod. Phys.* **D22** 1330018 (*arXiv:13037267* [astro-ph.CO]); E.I. Guendelman, H.Nishino, S. Rajpoot (2014) *Phys. Lett.* **B732** 156 (*arXiv:1403.4199* [hep-th]).

[3] E. Guendelman, R. Herrera, P. Labrana, E. Nissimov, S. Pacheva (2015) *Gen. Rel. Grav.* **47** art.10 (*arXiv:1408.5344v4* [gr-qc]); E. Guendelman, E. Nissimov, S. Pacheva (2015) in *Eight Mathematical Physics Meeting*, B. Dragovic and Z. Rakic (eds.), Belgrade Inst. Phys. Press (*arXiv:1407.6281v4* [hep-th]).

[4] A. Starobinsky (1980) *Phys. Lett.* **B91** 99-102.

[5] T. Chiba, T.Okabe, M. Yamaguchi (2000) *Phys. Rev.* **D62** 023511 (*arXiv:astro-ph/9912463*); C. Armendariz-Picon, V. Mukhanov, P. Steinhardt (2000) *Phys. Rev. Lett.* **85** 4438 (*arXiv:astro-ph/0004134*); C. Armendariz-Picon, V. Mukhanov, P. Steinhardt (2001) *Phys. Rev.* **D63** 103510 (*arXiv:astro-ph/0006373*); T. Chiba (2002) *Phys. Rev.* **D66** 063514 (*arXiv:astro-ph/0206298*).

[6] P.J.E. Peebles, A.Vilenkin (1999) *Phys. Rev.* **D59** 063505; S.A. Appleby, R.A. Battye, A.A. Starobinsky (2010) *JCAP* **1006** 005 (*arXiv:0909.1737* [astro-ph]); for a recent account see Md. Wali Hossain, R. Myrzakulov, M. Sami, E.N. Saridakis (2014) *Phys. Rev.* **D90** 023512 (*arXiv:1402.6661* [gr-qc]).

[7] N. Arkani-Hamed, L.J. Hall, C. Kolda, H. Murayama (2000) *Phys. Rev. Lett.* **85** 4434-4437 (*astro-ph/0005111*).

[8] R. Adam *et al.* (Planck Collaboration) (2015) *Astronomy & Astrophysics* http://dx.doi.org/10.1051/0004-6361/201425034 (*arXiv:1409.5738* [astro-ph.CO]).

[9] G.F.R. Ellis, R. Maartens (2004) *Class. Quant. Grav.* **21** 223 (*gr-qc/0211082*); G.F.R. Ellis, J. Murugan, C.G. Tsagas (2004) *Class. Quant. Grav.* **21** 233 (*arXiv:gr-qc/0307112*); D.J. Mulryne, R. Tavakol, J.E. Lidsey, G.F.R. Ellis (2005) *Phys. Rev.* **D71** 123512 (*arXiv:astro-ph/0502589*).

[10] S. Weinberg (1972) *"Gravitation and Cosmology – Principles and Applications of the General Theory of Relativity"*, John Wiley & Sons, Inc.

[11] E. Guendelman, E. Nissimov, S. Pacheva, M. Vasihoun (2014) *Bulg. J. Phys.* **41** 123-129 (*arXiv:1404.4733* [hep-th]); E. Guendelman, E. Nissimov, S. Pacheva, M. Vasihoun (2015) in *Eight Mathematical Physics Meeting*, B. Dragovic and Z. Rakic (eds.), Belgrade Inst. Phys. Press (*arXiv:1501.05518*)

[12] Adam G. Riess *et.al.* (Supernova Search Team) (1998) *Astronomical Journal* **116** 1009-1038 (*arxiv:astro-ph/9805201*); S. Perlmutter *et.al.* (The Supernova Cosmology Project) (1999) *Astrophysical Journal* **517** 565-586 (*arxiv:astro-ph/9812133*); A.G. Riess *et.al.* (2004) *Astrophysical Journal* **607** 665-687 (*arxiv:astro-ph/0402512*).

[13] E. Guendelman, E. Nissimov, S. Pacheva (2015) *arxiv:1504.01031.*